A Capillary Pressure-Driven Empirical Model for Permeability Estimation in Carbonate Reservoirs

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Abstract

Permeability prediction is a crucial aspect of reservoir characterization, typically derived from core analysis. Using mercury injection test data, permeability can also be predicted. Various models have been proposed for permeability estimation, with their coefficients depending on the pore geometry, rock heterogeneity, and pore throat size. Most existing models rely on a single saturation point or parameter, such as 35% or 25% mercury saturation, or the weighted geometric mean of pore throats and porosity. This study introduces a new empirical model that combines multiple effective parameters to estimate permeability. A total of 50 carbonate samples were used to develop the model, with 20 additional samples, and log data used for verification. In this study, permeability ranges from 0.01 to 450 mD, and porosity ranges from 1% to 30%. Multiple linear regression was employed to establish a relationship between permeability, porosity, R_{35} (the radius corresponding to 35% mercury saturation), and Swanson's parameter (the ratio of S_b/P_{cmax} , where P_{cmax} is the capillary pressure). This model addresses potential errors in previous models by incorporating more comprehensive parameters. The model was verified using mercury injection test data from various wells and has demonstrated promising results.

Introduction

The extraction of subsurface oil and gas resources depends on several essential factors, such as porosity, permeability, relative permeability (RP), capillary pressure, and wettability, among others (Feng et al. 2021). The permeability of rock is closely associated with the distribution of pore throat sizes, making the mercury injection capillary pressure (MICP) curve a reliable tool for predicting permeability. The RP curve illustrates the relationship between the permeabilities of various fluid phases, including oil and water, within a porous medium. This relationship governs the movement of these phases through the reservoir's porous structure and fracture networks, playing a crucial role in enhancing the precision of reservoir simulation models (Wang et al. 2023). The RP curve plays a vital role in reservoir modeling, as it greatly influences history matching, the development and optimization of production strategies, and enhanced recovery. Therefore, it is essential to develop efficient and precise techniques for obtaining RP curves.

Various techniques have been employed to obtain RP curves, generally classified into direct and indirect methods. The direct approach involves conducting laboratory experiments on rock cores using either steady-state or unsteady-state measurement techniques (Swanson 1981; Pittman 1992; Dastidar et al. 2007; Krevor et al. 2012; Feng et al. 2018). One commonly used technique is mercury injection, where mercury is introduced into the microscopic pores of a porous medium under controlled pressure conditions, establishing a correlation between pressure and the volume of injected mercury. The RP curves derived from these experiments are influenced by the complex micro-pore structure of the medium. Due to the ease of data acquisition and the ability to analyze relatively large sample sizes, numerous researchers have developed RP models based on capillary pressure experiments (Purcell 1949; Burdine 1953; Corey 1954; Brooks and Corey 1966). Purcell (1949) introduced a

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permeability model based on the capillary pressure curve, if water flows through smaller capillary tubes while gas moves through larger ones, leading to a straightforward RP model. Expanding on Purcell's foundation, Burdine (1953), Corey (1954), and Brooks and Corey (1966) developed RP models that incorporate pore size distribution and tortuosity; however, these models do not account for the presence of an irreducible water film. The integration of percolation theory into RP calculations, first introduced by Helba et al. (1992), has since been adopted and refined by several researchers, including Salomao (1997), Dixit et al. (1998), Phirani et al. (2009), and Kadet and Galechyan (2014).

One of the primary challenges in this approach is accurately determining coordination numbers and pore fractions within network models. Currently, many permeability models rely on the MICP curve, which can be categorized into two main types (Comisky et al. 2007). The first category includes permeability models based on percolation theory, which assumes that flow paths in porous media can be represented by a single-scale aperture. Notable examples within this category are the Kozeny-Carman model (Schwartz et al. 1989; Bernabé and Maineult 2015), the Katz-Thompson models (Katz and Thompson 1986), and the Revil-Glover-Pezard-Zamora model (Glover et al. 2006). The second category includes permeability models based on Poiseuille's equation and Darcy's law, which conceptualize flow paths in porous media as a collection of capillary tubes.

Notable models in this category include the Purcell model (Purcell 1949; Zhang et al. 2017), the Thomeer model (Thomeer 1960 and 1983), the R₃₅ model (initially proposed by Winland and later reported by Kolodzie (1980)), the Swanson model (Swanson 1981; Kamath 1992), the R₂₅ model (Pittman 1992), the Capillary-Parachor model (Guo et al. 2004; Liu et al. 2016; Xiao et al. 2017), the Huet model (Huet et al. 2005), the R₅₀ model (Rezaee et al. 2006; Gao and Hu 2013), and the R_{WGM} model (Dastidar et al. 2007), where R_{WGM} represents the weighted geometric mean radius. Zhou et al. (2023) applied the ensemble Kalman method to predict RP curves using saturation data, while Lanetc et al. (2024) developed a novel approach that integrates hybrid pore network and fluid volume methods for RP curve estimation. Additionally, Rezaei et al. (2020) focused on modifying permeability models initially designed for sandstones to enhance their applicability to carbonate reservoirs. While these studies have made notable progress in acquiring RP curves through various methodologies, each approach presents certain limitations. Therefore, the development of a more efficient framework for obtaining RP curves remains a critical objective. Various permeability models, including those developed by Winland (1992), Swanson (1981), and Dastidar (2007), have utilized different parameters to predict permeability, often calibrated using clastic or carbonate rock samples. Carbonate rocks, due to their diverse depositional environments and complex diagenetic processes, pose significant challenges for permeability modeling. Earlier models, such as those by Winland, Pittman, and Swanson, were designed for specific facies and diagenetic conditions, incorporating factors like pore throat size, porosity, and Swanson's parameter—the maximum ratio of S_b/P_{cmax}.

Although these models have contributed to permeability predictions, they have sometimes exhibited inaccuracies when applied to certain carbonate samples (Nooruddin et al. 2014). To address these shortcomings, this study introduces a new model that integrates multiple key parameters to improve permeability estimation in carbonate rocks. The proposed model includes porosity, permeability, the pore-throat radius at 35% mercury saturation, and Swanson's parameter, offering a more comprehensive approach. By considering a wider range of influential factors, this model aims to enhance the accuracy and reliability of permeability predictions for complex carbonate reservoirs.

Materials and Methods

In this study, 70 core plug samples were used from three wells within a carbonate reservoir. All core plugs were one inch in diameter and two inches in length. During the mercury injection capillary pressure (MICP) test, mercury is injected into a sample under increasing pressure, and mercury saturation is plotted against pressure. The resulting curve is used to determine key petrophysical properties. Three well-established methods for permeability prediction, including the Winland, Pitman, and Dastidar models, were evaluated. The results from each model were compared to laboratory-measured permeability values. Extracted data include pore-throat sizes and porosity.

The MICP test was conducted on all plug samples, with porosity calculated from the volume of mercury injection. Permeability was measured using air, following Darcy's law, and ranged from 0.01 mD to 450 mD, with porosity values between 1% and 30%. A multiple linear modeling approach was applied to propose an empirical relationship between permeability and MICP data. Linear regression, combined with actual permeability data, was employed to refine permeability prediction models. This method quantifies the relationship between key variables and permeability, ensuring simplicity and interpretability of the model. Incorporating multiple predictors and validating the model against real data enhances its accuracy and reliability.

The predicted and actual permeability values were then compared using the linear modeling approach. Additionally, a total of 1,367 thin sections were prepared to study the geological properties of the formations. A permeability log, generated from Stoneley waves, and 20 modular formation dynamic tests (MDT) were also used to validate the results.

Results

MICP Test. The mercury injection capillary pressure (MICP) test was used to extract pore-throat size distribution (PSTD), while also determining the porosity and permeability of the samples. Petrographical analysis revealed that the samples predominantly consist of grainstone, with occasional occurrences of mudstone, wackestone, and rare packstone (**Figure 1**). Samples for the MICP test were selected based on this distribution shown in Figure 1. Anhydrite observed in some samples; however, these were excluded from the study due to their lack of reservoir interest.



Figure 1—Sedimentary facies in the studied carbonate reservoir.

Winland Permeability Model. Winland established an empirical relationship between porosity, permeability, and the diameter of pore throats, expressed as follows (Kolodzie 1980):

where R_{35} is the radius of the pore-throat in 35 % of mercury saturation, K is permeability (mD), and φ is porosity (%). The correlation between predicted and measured permeabilities can be seen in **Figure 2(a)**.

Pittman Permeability Model. Pitman permeability model has been constructed and calibrated based as follows (Pittman et al. 1992):

$$\log K = -1.221 + 1.415 (\log \varphi) + 1.512 (\log R_{25})....(2)$$

where R_{25} is the radius of the pore-throat in 25 % of mercury saturation, K is permeability (mD), and φ is porosity (%). The correlation and coefficient of determination (R^2) between the measured and predicted permeabilities are presented in **Figure 2(b)**.

Dastidar Permeability Model. Dastidar permeability model uses R_{WGM} (weighted geometric mean of the pore-throat) and porosity (Dastidar et al. 2007) according to the following model:

 $\log k = -2.51 + 3.06 (\log \varphi) + 1.641 (\log R_{WGM})....(3)$

where K is permeability (mD), φ is porosity (%), and R_{WGM} is weighted geometric mean of the pore-throat radius (µm). The predicted and the measured permeabilities and their linear modeling are shown in **Figure 2(c)**.

Model Development and Validation. The development of the new permeability prediction model was guided by a thorough review of existing models, including those proposed by Swanson (1981), Pittman et al. (1992), and Winland (Kolodzie 980). Key parameters, characterized by significant coefficients and substantial geological influence on permeability, were prioritized. After integrating these factors, the model underwent rigorous testing to optimize its accuracy. The result is the refined permeability prediction model presented in this study.

Proposed Model. Based on samples from carbonate formations and using multiple linear modeling analysis, a new model is introduced here. This model incorporates a comprehensive set of influential factors for permeability estimation in carbonate reservoirs, where grainstone predominates, along with the presence of mudstone. The model is calibrated for a permeability range up to 90 mD. The advantage of this model lies in its integration of various criteria and factors, offering improvements over previously proposed models. The model was developed using multiple linear modeling analysis and is presented as follows:

$$K = 0.242 - 19.552 (\log \varphi) - 17.432 (\log R_{35}) + 3.123 (\frac{Sb}{P_{cmax}}), \dots (4)$$

where R_{35} is the radius of the pore-throat related to the 35 % of mercury saturation, K is permeability (mD), φ is porosity (%), and S_b/P_{cmax} is the maximum value of S_b/P_c (A point is Swanson's parameter).

The predicted values of permeability vs. the actual permeabilities and their linear modeling models are presented in **Figure 2(d)**. An R^2 value of 0.92 in the linear regression model indicates that 92% of the variability in the dependent variable is explained by the independent variable. The equation Y=1.46x-15.40 suggests a strong positive relationship between the variables. While this high R^2 suggests a good fit, it's essential to also consider residual patterns and the statistical significance of coefficients to ensure model robustness and avoid potential overfitting.



Figure 2—The measured permeabilities vs. their predicted values in Winland (a), Pitman (b), Dastidar (c), and the proposed model (d). The R² values and the slope of the lines and y-intercepts are also presented in each plot.

Verification of the Model. MICP data from two additional wells were used to verify the new model. The samples for verification were from the same carbonate formations. Predicted permeability values were compared with the measured values, yielding satisfactory results. These results are presented in **Figure 3**.



Figure 3—Comparison of measured and predicted permeability values based on the new model.

Model Verification Using Sonic Log and Stoneley Permeability. Permeabilities were also obtained using Stoneley waves, extracted from a sonic scanner in the reservoir. The permeability values derived from Stoneley waves showed a strong correlation with those obtained from modular formation dynamic tests (MDT). The results are presented in **Figure 4**.



Figure 4—Comparison of the generated permeability from Stoneley waves and MDT.

The next step involved comparing the permeability log, confirmed by MDT, with the predicted permeability from the proposed model. The result was satisfactory, with a R^2 value of 0.71, as shown in **Figure 5**.



Figure 5—Comparison of permeability derived from Stoneley waves and predicted permeability from the proposed model.

Discussion

This study proposes a new empirical model for permeability prediction in carbonate rocks. By considering the heterogeneity of various carbonate facies, the model achieves more accurate permeability predictions. Each experimental permeability model relies on core samples for calibration, and previous models, such as Winland's, used both carbonate and clastic formations for this process. Due to the differing petrophysical properties between carbonate and clastic rocks, permeability predictions from these models may not be fully accurate. As shown in Rezaei et al. (2024), variations in Mg/Ca concentrations were observed across parallel calcite crystal faces. Crystallographic orientation appears may influence the incorporation of impurities into minerals, affecting their physical and chemical properties (Rezaei 2023). Due to the diverse depositional environments and complex diagenetic processes of carbonate rocks, permeability modeling remains a significant challenge. Gabitov et al. (2022) emphasized the heterogeneous distribution of trace elements in carbonates, which impacts pore structure and complicates permeability estimation. These variations can affect calcite solubility, which in turn influences permeability, and adds complexity to reservoir prediction. Winland's model, averaged petrophysical features from both rock types, leading to potential inaccuracies, as evidenced by the R² values in Figure 2. Similarly, the Pitman and Dastidar models were primarily calibrated using clastic rocks, making them less reliable for carbonate reservoirs.

In contrast, the proposed model accounts for the distinct characteristics of carbonate formations, considering the different facies and sedimentary environments that influence permeability. This approach is particularly important for carbonate reservoirs, where geological features can vary significantly. For example, grainstones and packstones dominate the facies in the studied carbonate formations, with dolomite and limestone as the primary lithologies.

Different facies exhibit varying petrophysical characteristics, which influence permeability prediction. However, the proposed model incorporates samples from a range of facies with diverse petrophysical properties. Previous research (e.g., Nooruddin et al. 2016) has demonstrated that earlier models sometimes yield significant errors. In the current model, the linear regression equation between predicted and measured permeability is characterized by a slope and an R^2 value. A slope and R^2 value of 1 indicate a close match between actual and predicted permeability.

In the proposed model, the slope and R^2 values are 1.4 and 0.91, respectively. The new model incorporates more effective parameters, reducing the impact of errors in varying conditions. Key factors such as R_{35} , porosity, and S_b/P_{cmax} were specifically considered and adjusted for carbonate rocks. Other models were developed based on different formations, lithologies, and sedimentary environments. The permeability predictions from the proposed model, compared with actual permeability measurements, were reliable. Although the predictions were satisfactory, the model was further verified using data from different wells. The verification results showed a slope of 1.3 and an R^2 value of 0.85, indicating high accuracy in predicting permeability in these carbonate formations.

The model also demonstrated good agreement with modular formation dynamic tests (MDT), which reflect dynamic permeability under natural reservoir conditions. The comparison between the predicted permeability from the new model and the permeability log derived from Stoneley waves showed a strong correlation. These results indicate that accurate permeability data can be obtained under natural reservoir conditions.

Conclusions

This study presents a new empirical correlation for estimating permeability in carbonate reservoirs. The proposed model, developed using data from various carbonate formations, incorporates more effective parameters, resulting in improved permeability prediction. The model was applied to new data from two additional wells, yielding satisfactory results during verification. In practical applications, this correlation proves useful for predicting permeability in similar geological reservoirs. Future studies could explore the applicability of this model to other formations or investigate the effects of different parameter combinations.

Conflicting Interests

The author(s) declare that they have no conflicting interests.

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