An Analytical Equation to Predict Oil-Gas-Water Three-Phase Relative Permeability Curves in Fractures

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Abstract

As fractures are the major flow channels for multiphase flow in naturally and hydraulically fractured reservoirs, the accurate prediction of multiphase flow in fractures is highly important. The oil-gas-water three-phase relative permeability relations in fractures define the hydrodynamics of multiphase fluids flow and are necessary for modeling of multi-phase flow in fractured reservoirs.

In this work, a novel flow model based on the concept of shell momentum balance, Newton's law of viscosity, and the cubic law, is derived to determine analytic functions for the three-phase relative permeability curves versus phase saturation and viscosity in a single fracture.

The results show that the equations describing three-phase relative permeability curves in a fracture are function of saturations and viscosities. Water phase relative permeability depends on water saturation, gas phase relative permeability depends on gas saturation when μg is much lower than μo and μw. However, oil phase relative permeability is function of all-phase saturations. The isoperms of water phase and gas phase are straight lines. However, oil phase isoperms are functions of all phase saturations and have significant curvature. The curvatures of oil phase isoperms increase with the increase of μo. Gas saturation decreases oil phase relative permeability with a given oil saturation, while the viscosity ratio increases it.

Introduction

Multiphase flow in naturally fractured reservoirs and hydraulically fractured reservoirs, which holds major part of the world's remaining hydrocarbon reservoirs, is strongly influenced by fractures in the geological formations (Lei et al. 2014). Fractures are the major flow channels for fluid flowing in fractured reservoirs. Thus, it is highly important to predict multiphase flow in fractures accurately.

The three-phase relative permeability relations for fractures defines hydrodynamics of fluid flow and are necessary for modeling of multiphase flow in reservoirs.

The study of three-phase relative permeability was reported as early as 1941 by Leverett and Lewis (1941). They conducted steady-state three-phase relative permeability measurements in a tightly packed sand core. Corey et al. (1956) reported results of three-phase relative permeability measurements in Berea sandstone and proposed a model for prediction of three-phase relative permeability with assuming that the oil relative permeability depends on two saturations due to the dependence of residual oil saturation on two saturations. They suggested that the water phase isoperms and gas phase isoperms were straight lines. Sarem (1966) modified three-phase relative permeability measuring techniques by using unsteady-state technique. Donaldson and Dean (1965) used Sarem’s technique to measure three-phase relative

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permeability in Berea sandstone. Saraf and Fatt (1967) developed a new technique using NMR for in-situ saturation measurements in three-phase flow system. Stone (1970) proposed the Stone I model for prediction of three-phase relative permeability. Stone (1973) proposed the Stone II model using four two-phase flow relative permeability curves (two oil-water and two oil-gas) for predicting three-phase relative permeability.

Dietrich and Bonder (1976) proposed a model accounted for reduction in oil relative permeability due to the presence of a third phase. Spronsen (1982) measured a three-phase system in Berea sandstone using the centrifuge method. Saraf et al. (1982), Grader and O'Meara (1988) and Maini et al. (1990) measured the three-phase relative permeability using steady-state and unsteady-state methods. In addition, there have been more models (Maini et al. 1989; Hustad and Hansen 1995; Oosrom and Lenhard 1998; Balbinski et al. 1999) proposed for prediction of three-phase relative permeability. Oak et al. (1990) and Oak (1990) conducted a three-phase relative permeability measurement on water-wet fired Berea sandstone core and presented about 1,800 data collected for three-phase measurements for different saturation paths to investigate the effect of saturation history on relative permeability curves. These studies have great significance for studying three-phase relative permeability. However, these studies are for multiphase flow in porous media but not for fracture systems.

Compared with the studies on relative permeability in porous medium, relative permeability in fractures has received less attention. Many researchers have studied flow regimes in a fracture (Persoff et al. 1991; Persoff and Pruess 1995; Diomampo 2001; Fourar et al.1993; Fourar and Bories 1995; Pan 1999; Chen 2005) and revealed that the flow patterns not only depend on fracture geometry but also on phase properties. In order to examine flow of multiphase flow in fractures, some researchers have done different experimental studies (Chen et al. 2004; Chen 2005; Habana 2002; Kneafsy and Pruess 1998; Nicholl and Glass 1994; Nicholl et al. 2000; Pan 1999). The first relative permeability models for fracture systems were established by Romm (1966) based on experimental results using kerosene and water. Romm suggested that two-phase flow in fractures can be modeled by straight-line. However, many scholars (McDonald et al. 1991; Pieters and Graves 1994; Fourar and Bories 1995; Diomampo 2001; Speyer et al. 2007) had proved that relative permeability curves in fractures were not a simple linear function of saturation with experimental evidence. Many different theoretical studies have been conducted to examine multiphase flow in fractures (Bodin et al. 2003; Iwai 1976; Reis 1990; Shad and Gates 2010; Chima et al. 2010; Chima and Geiger 2012). Shad and Gate (2010) developed a new model for multiple-phase layer flow in a single fracture and concluded how relative permeability in fractures depended on flow structures as well as fluid properties. Chima et al. (2010) derived an analytical equation to calculate oil-water relative permeability curves in fracture systems. And they concluded that relative permeability curves in two-phase flow in fractures are not a linear function of saturation. Chima and Geiger (2012) presented a new model to predict gas-water relative permeability curves in a fracture. The model was validated with experimental data and showed much better agreement than original models. Although these experimental and theoretical studies relative permeability in a fracture and the influence of flow structures and fluid properties on relative permeability, they are not for oil-gas-water three-phase relative permeability. Based on Chima and Geiger's (2012) two-phase models, a novel model that can predict three-phase relative permeability in fracture systems was proposed in this paper. Although our three-phase relative permeability model is for fracture systems and different from previous models, the results of our studies match the previous studies (Corey et al. 1956; Donaldson and Dean 1966; Saraf et al. 1982; Oak 1990; and Maini et al. 1990).

Mathematical Model

The following assumptions are made to derive the proposed mathematical model of oil-gas-water relative permeability curves in the fracture:

1. Flow is laminar and in steady-state;
2. Fluids are Newtonian. Gas is compressible, oil and water are slightly compressible, all the phases have constant properties;
3. No phase transformation between the three phases;
4. Fracture walls are planar and impermeable, e.g. no fluid is exchanged between matrix and fracture;
5. The wettability of fracture walls sequences from water>oil>gas. Water phase flow occurs close to fracture surface, gas phase flows in the center of the fracture, oil phase flow occurs in-between water phase and gas phase;
6. Flow occurs in an open fracture, e.g. the planar surfaces representing the fracture walls remain parallel and thus are not in contact at any point;
7. The fracture is oriented horizontally and gravity is negligible.

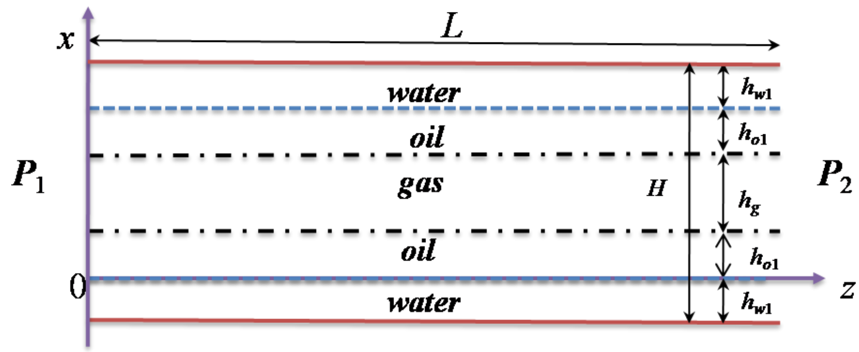


Figure 1—Proposed fracture model used in the mathematical model. For a perfectly smooth fracture placed horizontally with negligible gravity and buoyancy effects.

Applying the shell momentum balance to the fracture configuration shown in **Figure 1** leads to the following equations that allow reservoir engineers to estimate oil-gas-water relative permeability curves in fractures.

The detailed mathematical derivation is given in Appendix A.

$$k_{rw} = \frac{S_w^3}{2}(3 - S_w), \quad (1)$$

$$k_{rg} = S_g^2\left(S_g^2 + \frac{\mu_g}{\mu_w}\left(3S_w - \frac{3}{2}S_w^2\right) + \frac{\mu_g}{\mu_o}\left(3S_gS_o + \frac{3}{2}S_o^2\right)\right), \quad (2)$$

$$k_{ro} = S_o^2\left(\frac{\mu_o}{\mu_w}\left(3S_w - \frac{3}{2}S_w^2\right) + \frac{3}{2}S_gS_o + S_o^2\right), \quad (3)$$

Eqs. 1 through **3** are applied to water phase, gas phase, and oil phase, respectively. Eq. 1 implies that water relative permeability is only function of its saturation. Eq. 2 illustrates that gas relative permeability depends on all other phases' saturation. Eq. 3 reveals that oil relative permeability is not only function of saturation but also depends on water and gas saturations and oil-water two-phase viscosities. However, if μg is much lower than μ_w and μ_o, Eq. 2 can be simplified as,

$$k_{rg} = S_g^4, \quad (4)$$

Eq. 4 illustrates that gas relative permeability is only function of gas saturation when gas viscosity is much lower than all other phases' viscosity.

Model Validation and Model Analysis

With the basic parameters (**Table 1**) applied in the novel model, the oil-gas-water three-phase relative permeability curves in the fracture systems was estimated by the Eq. 1 to 3 and oil-gas-water three-phase isoperms of the study are given in **Figure 2**.

The results of this study confirm the dependency of water and gas relative permeabilities on their own saturations, and oil phase relative permeability to all the phases. The study also shows that the isoperms of water and gas phases are function of their own saturations. However, oil isoperms are not only function of oil saturation but also had significant curvature (concave towards the 100% oil saturation) which has the same conclusions with the previous studies (Corey et al. 1956; Donaldson and Dean 1966; Saraf et al. 1982; Oak 1990; and Maini et al. 1990).

Table 1-Basic parameters applied in the model.

Parameters	Value
Fracture length [m]	1.15
Fracture width [m]	2.25
Fracture thickness [mm]	0.75
Gas bed thickness [mm]	0.30
Water bed thickness [mm]	0.25
Oil bed thickness [mm]	0.20
Inlet pressure [MPa]	1.50
Gas density [kg/m^3]	0.82
Water density [kg/m^3]	1000
Oil density [kg/m^3]	810
Gas viscosity [cp]	0.017
Water viscosity [cp]	1.0
Oil viscosity [cp]	4.5
Outlet pressure [MPa]	1.45

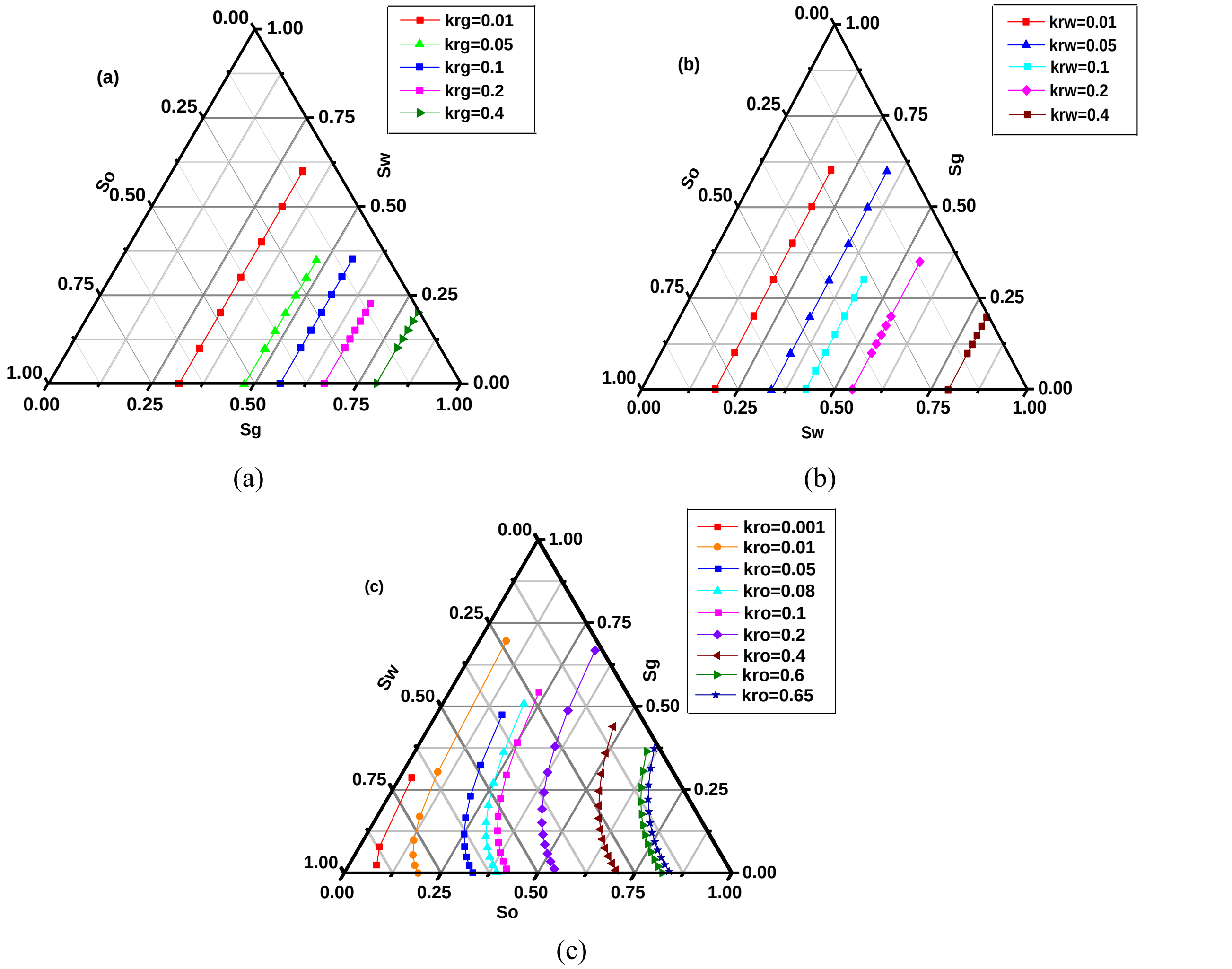


Figure 2—The isoperms of different phase for fracture systems calculated for the Example. (a) Gas phase. (b) Water phase. (c) Oil phase.

With the proposed model, series of three-phase relative permeability calculations at various oil viscosities were performed. The results show that the trend and curvature of oil isoperms varied with oil viscosity. The larger the viscosity of oil phase is, the greater the curvatures of oil isoperms are. Oil isoperms (k_{ro}=0.08) with different oil viscosities of the data sets are shown in **Figure 3**.

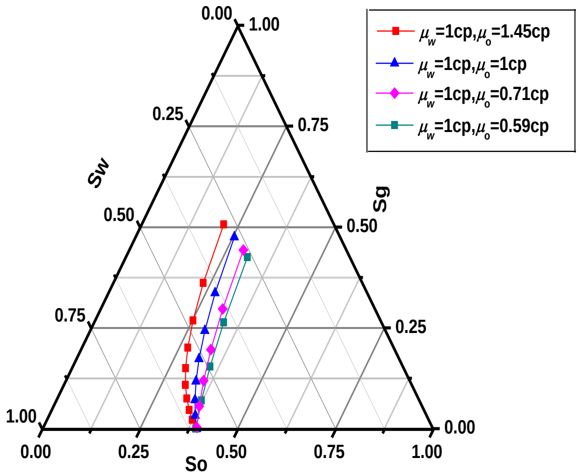


Figure 3—Oil isoperms for fracture systems with different viscosities of oil phase (k_{ro}=0.08).

Figure 4 shows the predicted oil phase relative permeability results at various water saturations. And oil viscosity is 1.45 cp. The results of this study show that oil phase relative permeability increased with the decrease of gas saturation (e.g. oil phase relative permeability increased with the increase of water saturation) with the same oil saturation. The study illustrates that gas saturation decreased oil relative permeability for the same oil saturation. The reason for this is that water phase flow occurs close to fracture surface, gas phase flows in the center of the fracture and oil phase flow occurs in-between water phase and gas phase. Under the same oil saturation, gas saturation decreases with the increase of water saturation. For the same thickness of oil bed, the larger water saturation is (e.g. the larger the thickness of water bed in the fracture is), the lower the thickness of gas bed is, the closer to the center of the fracture oil phase flow occurs and the faster oil phase flows in the fracture.

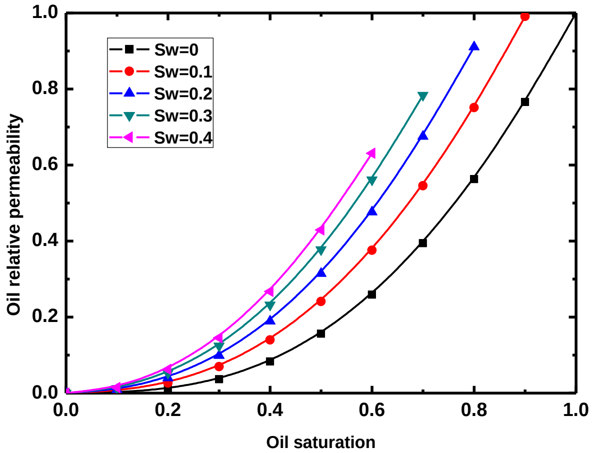
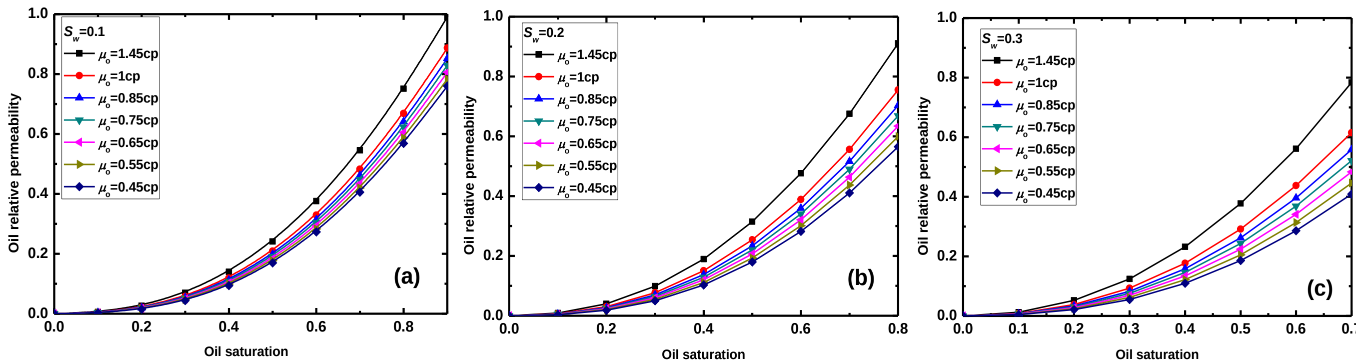


Figure 4—Oil relative permeabilities for fracture systems. The curves represent three-phase data at various water saturations when oil viscosity is 1.45 cp.

Figure 5 shows oil phase relative permeability results at various water saturations and viscosities of oil phase (as μ_w=1cp, μ_o represents viscosity ratio). The results show that oil phase relative permeability increases with the increase of viscosity ratio. The result can be explained as: wetting phase (water phase) flow occurs close to fracture surface, the flow of the adjacent high-viscosity non-wetting phase (oil phase) passing by it, to some extent, can be considered as a sliding motion in which the wetting phase (water phase) provides lubrication. Figure 5 also shows that water saturation intensifies the influence of viscosity ratio to oil relative permeability. The effect increases with the increase of water saturation.



(a) (b) (c)

Figure 5—Oil relative permeabilities for fracture systems. The curves represent three-phase data at various oil viscosities with different water saturations. (a) The value of water saturation is 0.1. (b) The value of water saturation is 0.2. (c) The value of water saturation is 0.3.

Conclusions

The following main conclusions can be drawn from this study:

1. A novel analytical model for oil-gas-water three-phase relative permeability in fracture systems has been proposed in this study. The equations describing oil-gas-water three-phase relative permeability curves in a fracture are function of saturations and viscosities.
2. The novel model was validated with the previous studies. The isoperms of water and gas phases are function of their own saturations, however, oil isoperms are not only function of oil saturation but had significant curvature.
3. The study illustrated that the trend and curvature of oil isoperms varied with oil viscosity. The curvatures of oil isoperms increase with the increase of the viscosity of oil phase. Gas saturation decreases oil relative permeability, however, the viscosity ratio increases it.

Conflicts of Interest

The author(s) declare that they have no conflicting interests.

Nomenclature

A_s	=	area of the fracture
A_w	=	area of water bed in the fracture
A_g	=	area of gas bed in the fracture
A_o	=	area of oil bed in the fracture
C_1	=	integration constant
C_{11}	=	integration constant
C_{12}	=	integration constant
C_{13}	=	integration constant
C_{14}	=	integration constant
C_{15}	=	integration constant
C_1^{wb}	=	integration constant of water phase at the bottom
C_1^{g}	=	integration constant of gas phase
C_1^{wt}	=	integration constant of water phase at the top
C_1^{ob}	=	integration constant of oil phase at the bottom
C_1^{ob}	=	integration constant of oil phase at the top
h_g	=	thickness of gas bed in the fracture
h_{w1}	=	thickness of water bed in the fracture
h_{o1}	=	thickness of oil bed in the fracture
k_{rw}	=	relative permeability to water phase
k_{rg}	=	relative permeability to gas phase
k_{ro}	=	relative permeability to oil phase
k_e	=	absolute permeability
L	=	fracture length
P_1	=	pressure in z=0
P_2	=	pressure in z=L
S_w	=	water saturation
S_g	=	gas saturation

S_o = oil saturation
v_z^{wb} = velocity of water phase at the bottom
v_z^{wt} = velocity of water phase at the top
v_z^{ob} = velocity of oil phase at the bottom
v_z^{ot} = velocity of oil phase at the top
v_z^{g} = velocity of gas phase
V_z^{w} = average velocity of water phase
V_z^{o} = average velocity of water phase
V_z^{g} = average velocity of gas phase
W = width of fracture
Δx = difference operator
μ_g = viscosity of gas
μ_o = viscosity of oil
μ_w = viscosity of water
τ_{xz}^{wb} = shear stress for the water phase at the bottom
τ_{xz}^{wt} = shear stress for the water phase at the top
τ_{xz}^{ob} = shear stress for the oil phase at the bottom
τ_{xz}^{ot} = shear stress for the oil phase at the top
τ_{xz}^{g} = shear stress for the gas phase

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Appendix A

The fracture geometry and flow configuration for the proposed model is shown in Figure 1. For the proposed model, the modeled fracture comprises two smooth and planar fracture walls. Within the fracture, oil-gas-water three fluids are flowing in a steady-state condition. The wettability of fracture walls sequences from water>oil>gas. Gas phase flows in the center of the fracture. Water phase flow occurs close to fracture surface. Oil phase flow occurs in-between water phase and gas phase.

Applying the shell momentum balance (Bird et al. 2002; Chima et al. 2010; Chima and Geiger 2012) in the fracture, the following equation can be written as,

$$\frac{\partial \tau_{xz}}{\partial x} = \frac{P_1 - P_2}{L}, \qquad \text{(A-1)}$$

With the integration of the **Eq. A-1** for the five regions, the shear stress for water phase at the bottom, oil phase at the bottom, gas phase, oil phase at the top and water phase at the top can be written as,

$$\tau_{xz}^{wb} = \frac{P_1 - P_2}{L} x + C_1^{wb}, \quad \text{(A-2)}$$

$$\tau_{xz}^{ob} = \frac{P_1 - P_2}{L} x + C_1^{ob}, \quad \text{(A-3)}$$

$$\tau_{xz}^{g} = \frac{P_1 - P_2}{L} x + C_1^{g}, \quad \text{(A-4)}$$

$$\tau_{xz}^{ot} = \frac{P_1 - P_2}{L} x + C_1^{ot}, \quad \text{(A-5)}$$

$$\tau_{xz}^{wt} = \frac{P_1 - P_2}{L} x + C_1^{wt}, \quad \text{(A-6)}$$

Based on the boundary conditions (Chima et al. 2010; Chima and Geiger 2012; Lei et al. 2014) in the fracture, the following equations can be written as,

Condition 1: at $x = 0, \tau_{xz}^{wb} = \tau_{xz}^{ob}; v_z^{wb} = v_z^{ob}$

Condition 2: at $x = h_{o1}, \tau_{xz}^{g} = \tau_{xz}^{ob}; v_z^{g} = v_z^{ob}$

Condition 3: at $x = h_{o1} + h_g, \tau_{xz}^{g} = \tau_{xz}^{ot}; v_z^{g} = v_z^{ot}$

Condition 4: at $x = 2h_{o1} + h_g, \tau_{xz}^{wt} = \tau_{xz}^{ot}, v_z^{wt} = v_z^{ot}$

Condition 5: at $x = -h_{w1},\ v_z^{wb} = 0$

Condition 6: at $x = h_{w1} + 2h_{o1} + h_g,\ v_z^{wt} = 0$

With the boundary conditions 1-4, we can find that $C_1^g = C_1^{wb} = C_1^{ob} = C_1^{ot} = C_1^{wt} = C_1$. When Newton's law of viscosity is substituted into **Eqs. A-2** through **A-6**, the velocity equations for water phase at the bottom, oil phase at the bottom, gas phase, oil phase at the top and water phase at the top can be written as,

$$v_z^{wb} = -\left(\frac{P_1 - P_2}{L}\frac{x^2}{2\mu_w} + \frac{C_1}{\mu_w}x\right) + C_{11}, \quad \text{(A-7)}$$

$$v_z^{ob} = -\left(\frac{P_1 - P_2}{L}\frac{x^2}{2\mu_o} + \frac{C_1}{\mu_o}x\right) + C_{12}, \quad \text{(A-8)}$$

$$v_z^{g} = -\left(\frac{P_1 - P_2}{L}\frac{x^2}{2\mu_g} + \frac{C_1}{\mu_g}x\right) + C_{13}, \quad \text{(A-9)}$$

$$v_z^{ot} = -\left(\frac{P_1 - P_2}{L}\frac{x^2}{2\mu_o} + \frac{C_1}{\mu_o}x\right) + C_{14}, \quad \text{(A-10)}$$

$$v_z^{wt} = -\left(\frac{P_1 - P_2}{L}\frac{x^2}{2\mu_w} + \frac{C_1}{\mu_w}x\right) + C_{15}, \quad \text{(A-11)}$$

With the boundary conditions 1-5, the velocity profiles for the oil-gas-water three-phase can be written as,

$$v_z^{wb} = -\frac{P_1 - P_2}{L}\frac{x^2}{2\mu_w} + \frac{P_1 - P_2}{L}\frac{h_g + 2h_{o1}}{2}\frac{x}{\mu_w} + \frac{P_1 - P_2}{L}\frac{h_{w1}^2 + h_{w1}h_g + 2h_{w1}h_{o1}}{2\mu_w}, \quad \text{(A-12)}$$

$$v_z^{ob} = -\frac{P_1 - P_2}{L}\frac{x^2}{2\mu_o} + \frac{P_1 - P_2}{L}\frac{h_g + 2h_{o1}}{2}\frac{x}{\mu_o} + \frac{P_1 - P_2}{L}\frac{h_{w1}^2 + h_{w1}h_g + 2h_{w1}h_{o1}}{2\mu_w}, \quad \text{(A-13)}$$

$$v_z^g = -\frac{x^2}{2\mu_g}\frac{P_1-P_2}{L} + \frac{h_g+2h_{o1}}{2}\frac{P_1-P_2}{L}\frac{x}{\mu_g} + \frac{P_1-P_2}{L}\left(\frac{h_{w1}^2+h_g h_{w1}+2h_{w1}h_{o1}}{2\mu_w} + \frac{h_g h_{o1}+h_{o1}^2}{2\mu_o} - \frac{h_g h_{o1}+h_{o1}^2}{2\mu_g}\right), \quad \text{(A-14)}$$

$$v_z^{ot} = -\frac{P_1-P_2}{L}\frac{x^2}{2\mu_o} + \frac{P_1-P_2}{L}\frac{h_g+2h_{o1}}{2}\frac{x}{\mu_o} + \frac{P_1-P_2}{L}\frac{h_{w1}^2+h_g h_{w1}+2h_{w1}h_{o1}}{2\mu_w}, \quad \text{(A-15)}$$

$$v_z^{wt} = -\frac{P_1-P_2}{L}\frac{x^2}{2\mu_w} + \frac{P_1-P_2}{L}\frac{h_g+2h_{o1}}{2}\frac{x}{\mu_w} + \frac{P_1-P_2}{L}\frac{h_{w1}^2+h_g h_{w1}+2h_{w1}h_{o1}}{2\mu_w}, \quad \text{(A-16)}$$

The average velocities for each phase are calculated as (Chima et al. 2010; Chima and Geiger 2012; Lei et al. 2014),

$$V_z^w = V_z^{wb} = V_z^{wt} = \frac{1}{h_{w1}}\int_{-h_{w1}}^{0} v_z^{wb}\partial x = \frac{P_1-P_2}{L}\frac{4h_{w1}^2+3h_{w1}h_g+6h_{w1}h_{o1}}{12\mu_w}, \quad \text{(A-17)}$$

$$V_z^o = V_z^{ob} = V_z^{ot} = \frac{1}{h_{o1}}\int_0^{h_{o1}} v_z^{ob}\,\partial x = \frac{P_1-P_2}{L}\left(\frac{h_{w1}^2+h_{w1}h_g+2h_{w1}h_{o1}}{2\mu_w} + \frac{3h_g h_{o1}+4h_{o1}^2}{12\mu_o}\right), \quad \text{(A-18)}$$

$$V_z^g = \frac{1}{h_g}\int_{h_{o1}}^{h_{o1}+h_g} v_z^g\,\partial x = \frac{P_1-P_2}{L}\left(\frac{h_g^2}{12\mu_g} + \frac{h_{w1}^2+h_g h_{w1}+2h_{w1}h_{o1}}{2\mu_w} + \frac{h_g h_{o1}+h_{o1}^2}{2\mu_o}\right), \quad \text{(A-19)}$$

With the Darcy law, the following equations for oil-gas-water three-phase can be written as,

$$\frac{k_{ro}k_e A_s}{\mu_o}\frac{\Delta P_o}{L} = A_o V_z^o, \quad \text{(A-20)}$$

$$\frac{k_{rg}k_e A_s}{\mu_g}\frac{\Delta P_g}{L} = A_g V_z^g, \quad \text{(A-21)}$$

$$\frac{k_{rw}k_e A_s}{\mu_w}\frac{\Delta P_w}{L} = A_w V_z^w, \quad \text{(A-22)}$$

Where (see Appendix B for details),

$$\Delta P_o = \frac{P_1-P_2}{S_o};\ \Delta P_g = \frac{P_1-P_2}{S_g};\ \Delta P_w = \frac{P_1-P_2}{S_w}$$

$$A_o = 2Wh_{o1}; h_{o1} = S_o H = (1-S_g-S_w)H$$

$$A_g = Wh_g;\ h_g = S_g H;\ H = h_g + 2h_{w1} + 2h_{o1}$$

$A_s = WH; A_w = 2Wh_{w1}; h_{w1} = \frac{S_w}{2}H; k_e = \frac{H^2}{12}$ (Snow 1965; Witherspoon et al. 1980; Golf-Racht 1982).

Rearranging Eqs. A20 to A22 give the relative permeability for oil-gas-water three-phase,

$$k_{ro} = S_o^2\left(\frac{\mu_o}{\mu_w}\left(3S_w - \frac{3}{2}S_w^2\right) + \frac{3}{2}S_g S_o + S_o^2\right), \quad \text{(A-23)}$$

$$k_{rg} = S_g^2\left(S_g^2 + \frac{\mu_g}{\mu_w}\left(3S_w - \frac{3}{2}S_w^2\right) + \frac{\mu_g}{\mu_o}\left(3S_g S_o + \frac{3}{2}S_o^2\right)\right), \quad \text{(A-24)}$$

$$k_{rw} = \frac{S_w^3}{2}(3-S_w), \quad \text{(A-25)}$$

Eqs. A-23 through A-25 are the proposed equations to estimate oil-gas-water relative permeability curves in fractures.

Appendix B

For the fracture geometry and flow configuration shown in Figure 1, the following equations can be obtained as,

$$h_{o1} = S_o \frac{H}{2}, \quad \text{(B-1)}$$

$$h_g = S_g H, \quad \text{(B-2)}$$

$$S_w = \frac{WL(2h_{w1})}{WLH} = \frac{2h_{w1}}{H}, \quad \text{(B-3)}$$

The equalities ΔP_o, ΔP_g and ΔP_w are given below (Chima et al. 2010; Chima and Geiger 2012; Lei et al. 2014),

$$\Delta P_o = \frac{P_1 - P_2}{S_o} = \frac{P_1 - P_2}{1 - S_w - S_g}, \quad \text{(B-4)}$$

$$\Delta P_g = \frac{P_1 - P_2}{S_g}, \quad \text{(B-5)}$$

$$\Delta P_w = P_{w1} - P_{w2} = \frac{P_1 - P_2}{S_w}, \quad \text{(B-6)}$$

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